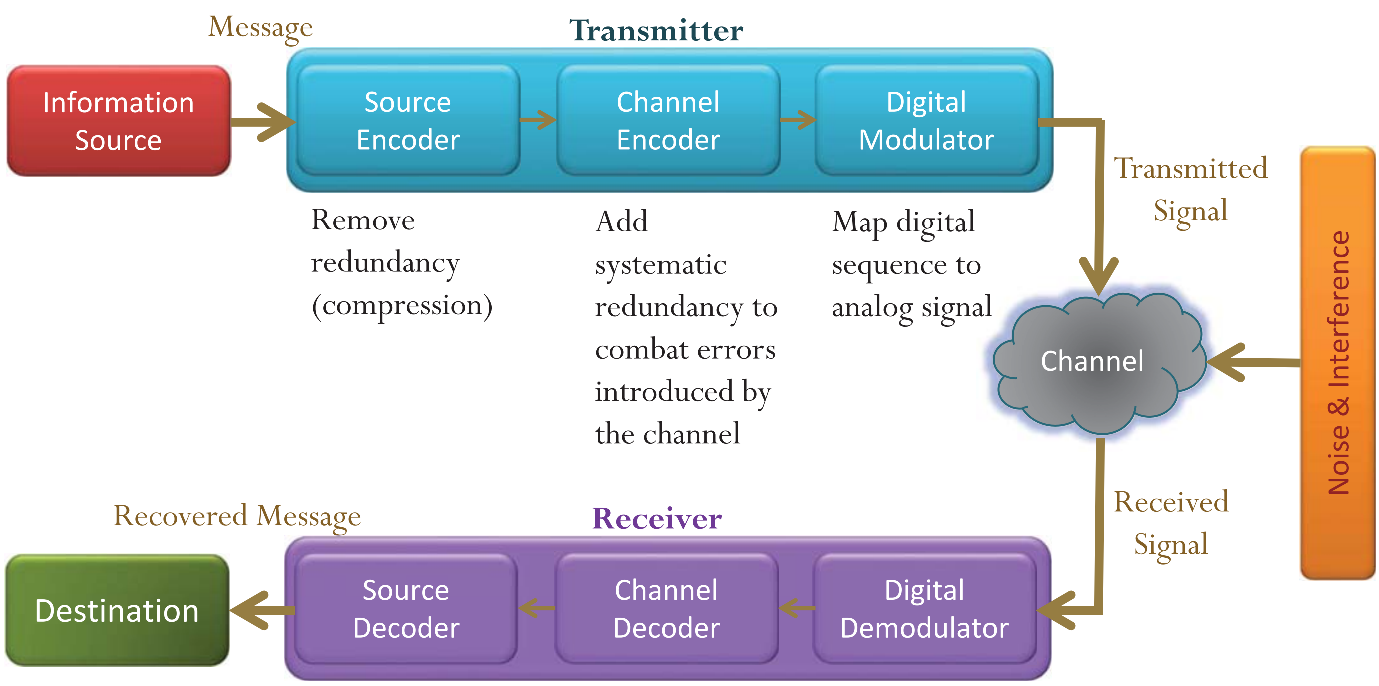
Digital Communication Systems
EES 452

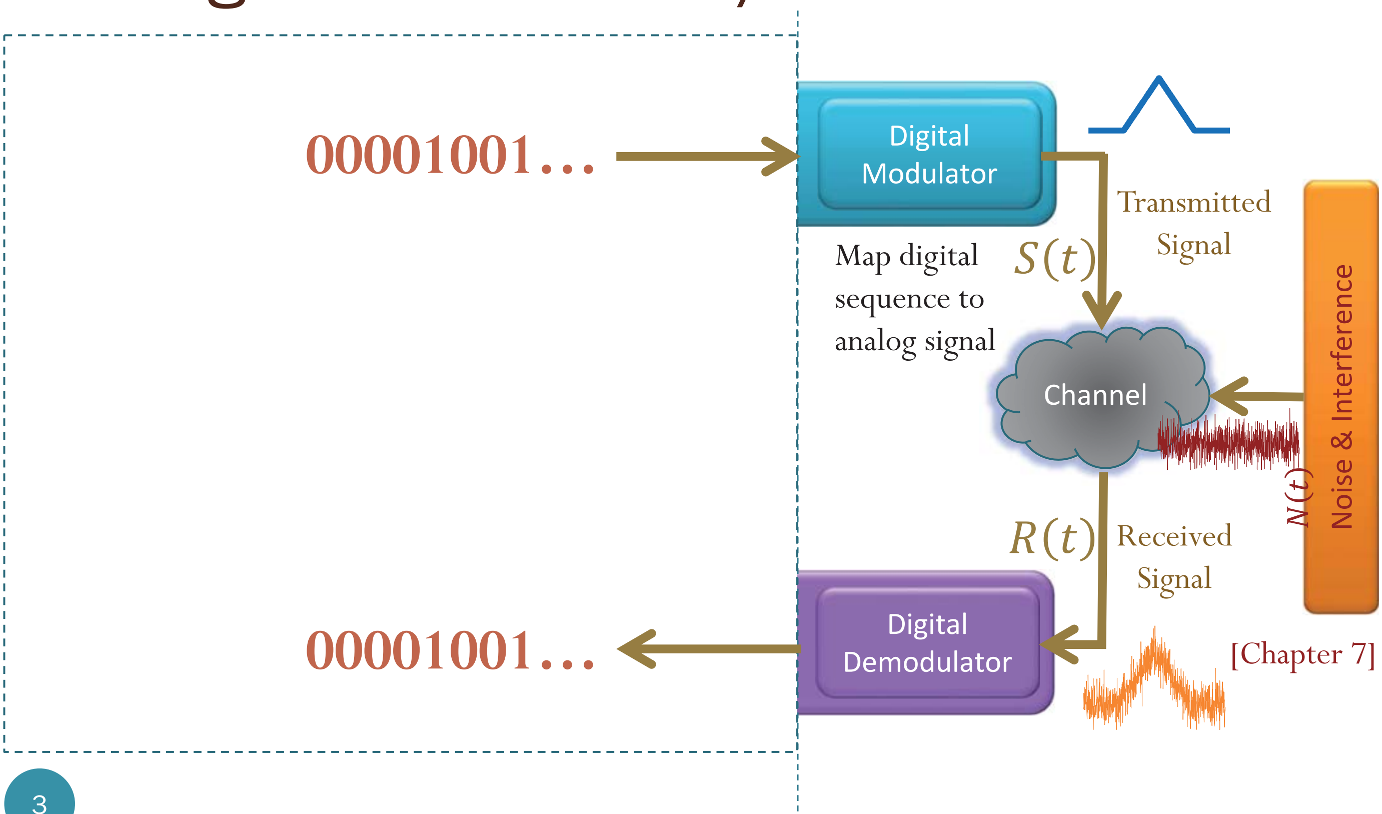
Asst. Prof. Dr. Prapun Suksompong
prapun@siit.tu.ac.th

6-7. Waveform Channel and The Conversion to Vector Channel

Elements of digital commu. sys.

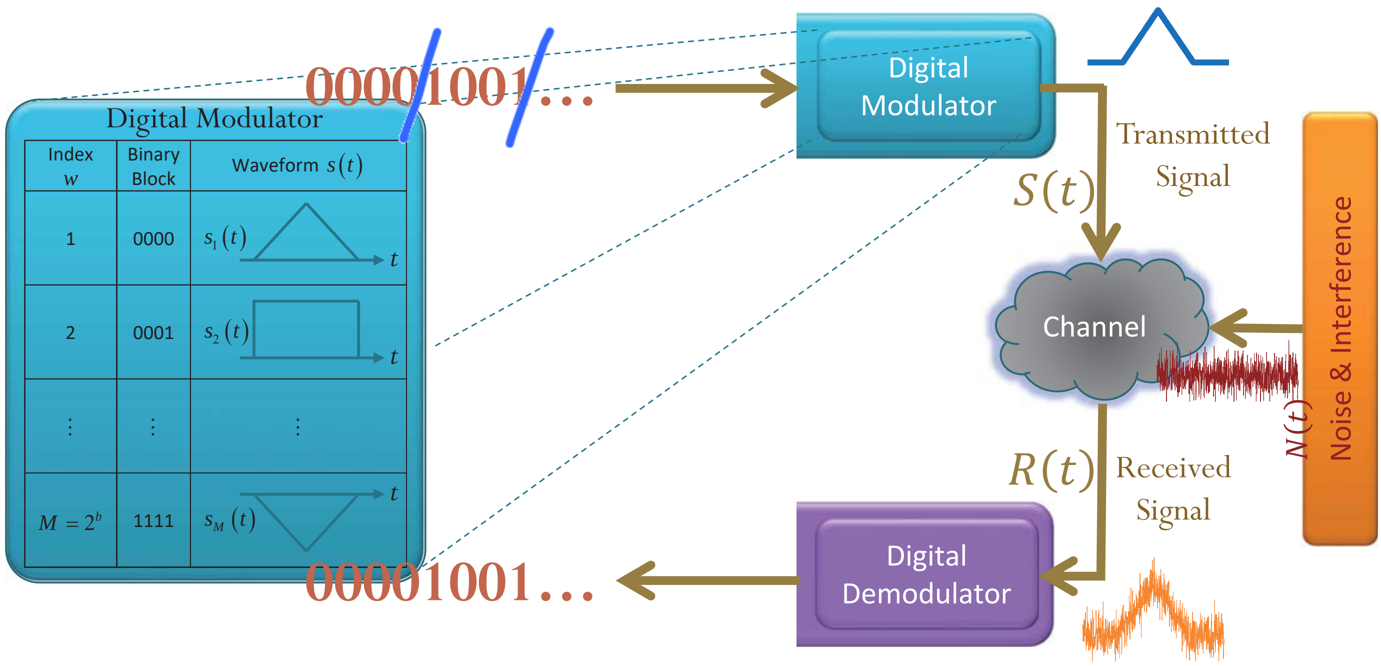


Digital Modulation/Demodulation

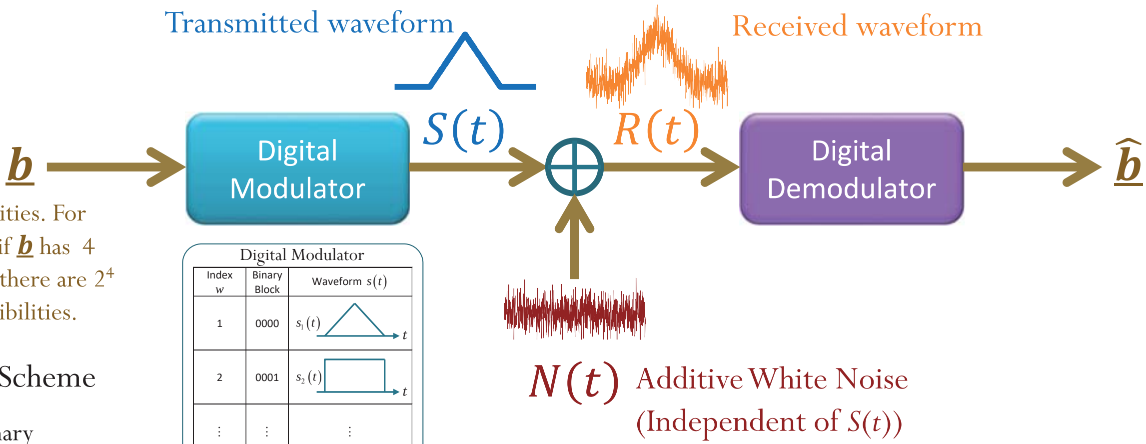


Digital Modem: Ex 1

$b = 4$: Work on 4-bit blocks.
Need $M = 2^b = 16$ different waveforms to represent different possibilities of the bit blocks.



Analysis of Digital Modem



M possibilities. For example, if b has 4 bits, then there are $2^4 = 16$ possibilities.

***M*-ary** Scheme

$M = 2$: Binary
$M = 3$: Ternary
$M = 4$: Quaternary

Digital Modulator

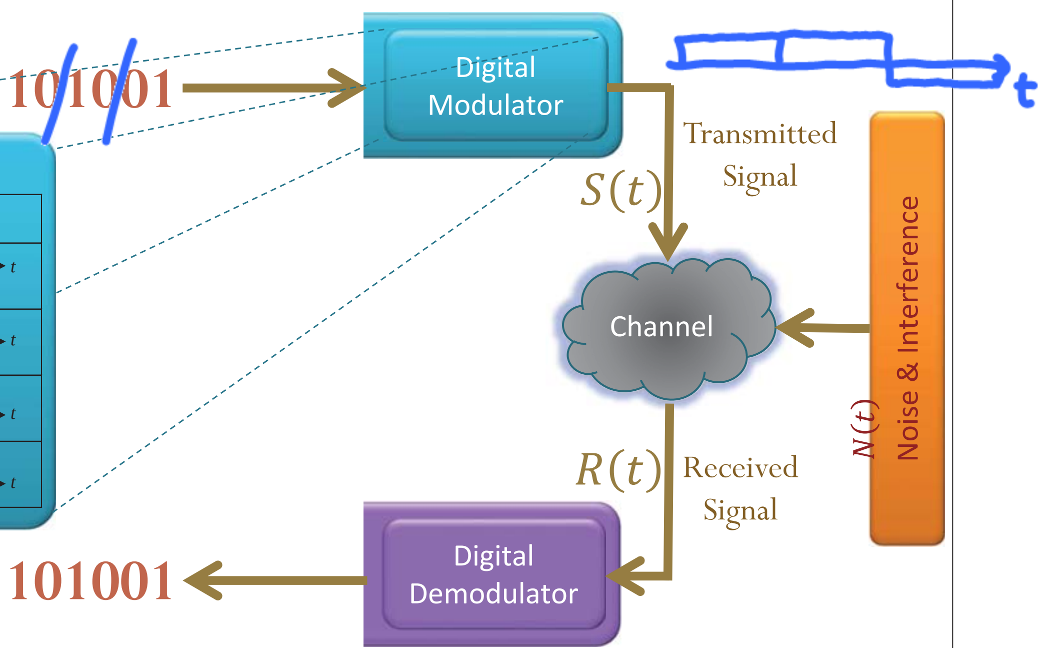
Index w	Binary Block	Waveform $s(t)$
1	0000	$s_1(t)$
2	0001	$s_2(t)$
⋮	⋮	⋮
$M = 2^b$	1111	$s_M(t)$

M possible "messages" requires
M possibilities for $S(t)$:

$$\{s_1(t), s_2(t), \ldots, s_M(t)\} \longleftarrow$$ We refer to this as the **signal set**

Digital Modem: Ex 2

$b = 2$: Work on 2-bit blocks. Need $M = 2^b = 4$ different waveforms to represent different possibilities of the bit blocks.



PAM Example: $M = 4$

Index m	Binary Block $\mathbf{b}$	Amplitude A_m	Waveform $s_m(t)$
1	00	-3	$s_1(t)$
2	01	-1	$s_2(t)$
3	10	1	$s_3(t)$
$M = 4$	11	3	$s_4(t)$

Pulse Amplitude Modulation (PAM)

- Use a common **pulse $p(t)$**.

Important special case of PAM

Amplitude-Shift Keying (ASK): $p(t) = g(t)\cos(2\pi f_c t)$

  - $E_p = \frac{1}{2} E_g$ when g is an energy signal that is bandlimited to $B < f_c$

- The pulse is scaled by M different "amplitudes":

$$\boxed{s_m(t) = A_m p(t), \quad 1 \le m \le M}$$

$p(t)$

PAM Example: $M = 2$

Index m	Binary Block **b**	Amplitude A_m	Waveform $s_m(t)$
1	0	-1	$s_1(t)$
$M = 2$	1	1	$s_2(t)$

PAM Example: $M = 4$

Index m	Binary Block **b**	Amplitude A_m	Waveform $s_m(t)$
1	00	-3	$s_1(t)$
2	01	-1	$s_2(t)$
3	10	1	$s_3(t)$
$M = 4$	11	3	$s_4(t)$

Analysis of Digital Modem

$\{s_1(t), s_2(t), \dots, s_M(t)\}$

Waveform Channel: $R(t) = S(t) + N(t)$

Find "orthonormal basis" unit energy

$\{\phi_1(t), \phi_2(t), \dots, \phi_K(t)\}$ → axes

orthogonal $\langle \phi_i, \phi_j \rangle = 0$ for $i \neq j$

Inner Product:
For two waveforms $x(t)$ and $y(t)$,
$\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t) y^*(t)\, dt$

$$\begin{pmatrix} \langle R(t), \phi_1(t) \rangle \\ \langle R(t), \phi_2(t) \rangle \\ \vdots \\ \langle R(t), \phi_K(t) \rangle \end{pmatrix} \quad \begin{pmatrix} \langle S(t), \phi_1(t) \rangle \\ \langle S(t), \phi_2(t) \rangle \\ \vdots \\ \langle S(t), \phi_K(t) \rangle \end{pmatrix} \quad \begin{pmatrix} \langle N(t), \phi_1(t) \rangle \\ \langle N(t), \phi_2(t) \rangle \\ \vdots \\ \langle N(t), \phi_K(t) \rangle \end{pmatrix}$$

Ex:
$K = 1$: PAM, ASK
$K = 2$: PSK, QAM

$$\langle x(t), x(t) \rangle = \int_{-\infty}^{\infty} x(t) x^*(t)\, dt = \int_{-\infty}^{\infty} |x(t)|^2 dt = E_x$$

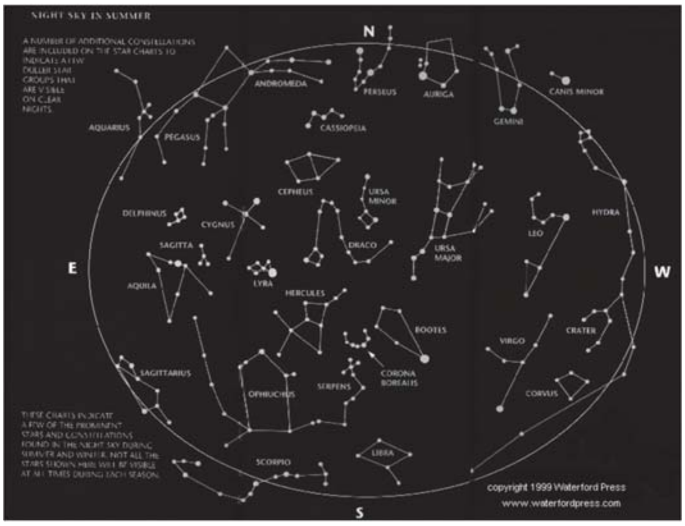
Vector Channel: $\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$

$\in \left\{ \vec{\mathbf{s}}^{(1)}, \vec{\mathbf{s}}^{(2)}, \dots, \vec{\mathbf{s}}^{(M)} \right\}$

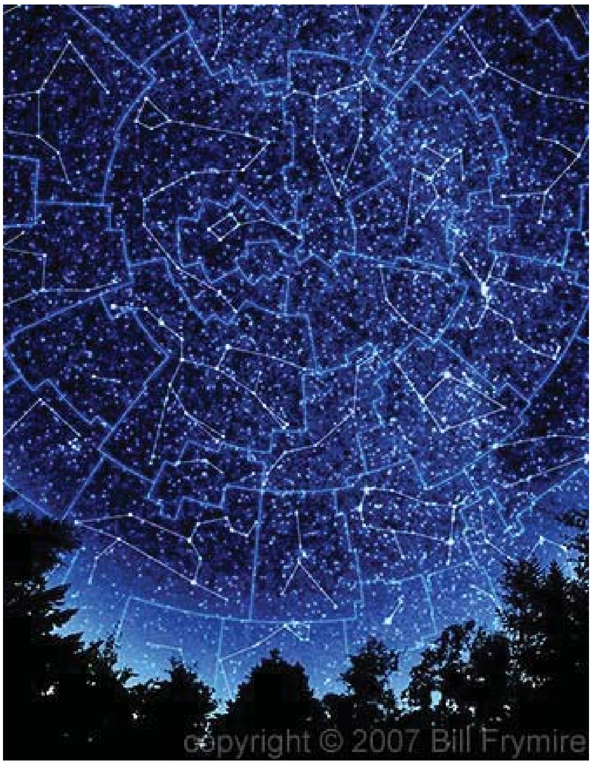
$\vec{\mathbf{R}}$, $\vec{\mathbf{S}}$ and $\vec{\mathbf{N}}$ are all random vectors.
Each vector contain K random variables.

can be visualized in the form of **signal constellation**

Star Constellations



[http://iamintellectuallypromiscuous.com/science/star-right-straight-morning/]



[http://68.media.tumblr.com/89eed4669ec511bbb6413acb8da8b0e2/tumblr_mz3qmyQeLH1rhb9f5o1_r1_400.jpg]

Star Constellations

constellation

noun [C]

UK /ˌkɒn.stəˈleɪ.ʃən/ **US** /ˌkɑːn.stəˈleɪ.ʃən/

any of the groups of stars in the sky that seem from earth to form a pattern and have been given names

sololos/E+/GettyImages

LONGDO Dict
บริการค้นหาและร่วมสร้างเนื้อหาพจนานุกรมหลายภาษา-ไทย

- English-Thai: NECTEC's Lexitron-2 Dictionary [with local updates]

constellation	[N] กลุ่ม
constellation	[N] กลุ่มดาว, **Syn.** group of stars, configuration of stars

- English-Thai: HOPE Dictionary [with local updates]

constellation	n. กลุ่มดาว, **See also:** constellatory adj.

- English-Thai: Nontri Dictionary

constellation	(n) หมู่ดาว,ดาวฤกษ์,ดารากร

- อังกฤษ-ไทย: คลังศัพท์ไทย โดย สวทช.

constellation	กลุ่มดาว, กลุ่มของดาวฤกษ์ที่ปรากฏในท้องฟ้า เคลื่อนที่ไปพร้อมกันเป็นกลุ่ม ๆ เช่นกลุ่มดาวจระเข้ กลุ่มดาวเต่า เป็นต้น [พจนานุกรมศัพท์ สสวท.]

https://dictionary.cambridge.org/dictionary/english/constellation
https://dict.longdo.com/search/constellation

From $s(t)$ to $\vec{\mathbf{s}}$

- Orthonormal basis $\{\phi_1(t), \phi_2(t), \dots, \phi_K(t)\}$. ⇒ axes

$$\langle \phi_i(t), \phi_j(t) \rangle = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

- Suppose we can express a waveform as a linear combination of the ϕ_{i}:

$$s(t) = c_1\phi_1(t) + c_2\phi_2(t) + \cdots + c_K\phi_K(t)$$

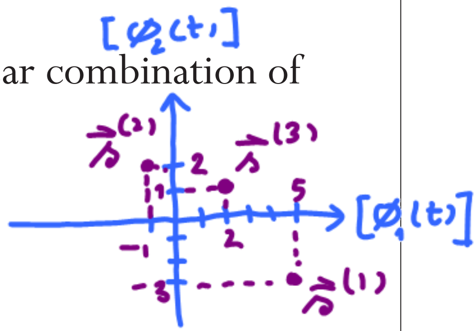
- Then, the vector representation of this waveform can be found easily from the coefficients in the linear combination:

$$s(t) = c_1\phi_1(t) + c_2\phi_2(t) + \cdots + c_K\phi_K(t) \rightarrow \vec{\mathbf{s}} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_K \end{pmatrix}$$

Example: From $s(t)$ to $\vec{\mathbf{s}}$

- Orthonormal basis: $\{\phi_1(t), \phi_2(t)\}$ ⇒ axes
- Suppose we can express the waveforms as a linear combination of the ϕ_{i}:

$$s_1(t) = 5\phi_1(t) - 3\phi_2(t)$$
$$s_2(t) = -\phi_1(t) + 2\phi_2(t)$$
$$s_3(t) = 2\phi_1(t) + \phi_2(t)$$



- Then, the vector representations of the waveforms can be found easily:

$$s_1(t) = 5\phi_1(t) - 3\phi_2(t) \rightarrow \vec{\mathbf{s}}^{(1)} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$s_2(t) = -\phi_1(t) + 2\phi_2(t) \rightarrow \vec{\mathbf{s}}^{(2)} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$s_3(t) = 2\phi_1(t) + \phi_2(t) \rightarrow \vec{\mathbf{s}}^{(3)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

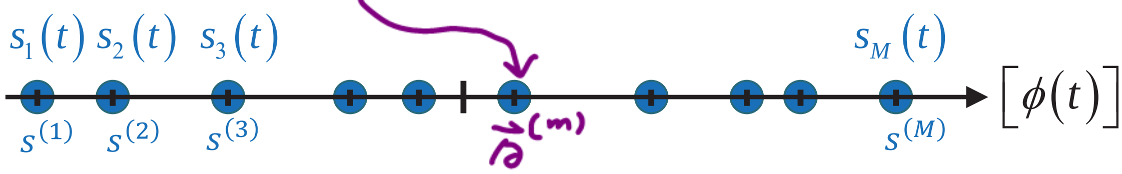
Constellation for PAM

- Recall that, for PAM,

$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

- Let $\phi(t) = \frac{p(t)}{\sqrt{E_p}}$ where $E_p = \int_{-\infty}^{\infty} |p(t)|^2 \, dt$.
  - Note that $E_\phi = \int_{-\infty}^{\infty} |\phi(t)|^2 \, dt = 1$.
- Then,

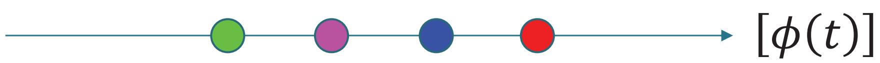
$$s_m(t) = \underbrace{A_m \sqrt{E_p}}_{} \phi(t) = s^{(m)} \phi(t), \quad 1 \le m \le M$$



PAM in the Vector Channel

$$\boxed{R = S + N}$$

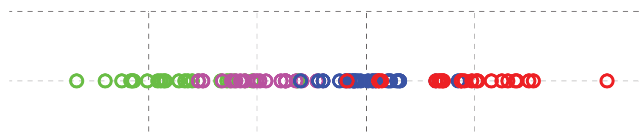
@Tx



The points are chosen randomly (according to the bits that are fed into the digital modulator) 100 times.

Add noise

@ Rx



Phase-Shift Keying (PSK)

- Digital phase modulation

$$\boxed{s_m(t) = g(t)\,cos(2\pi f_c t + \theta_m), \quad m = 1,2,\dots,M}$$

  - $g(t)$ is the signal pulse shape
  - $\theta_m = \frac{2\pi}{M}(m-1)$ represents phase of the carrier that convey the transmitted information.

$cos(A+B) = cosA\,cosB - sinA\,sinB$

- Let

$$\phi_1(t) = \sqrt{\frac{2}{E_g}}\,g(t)\,cos(2\pi f_c t) \;\text{ and }\; \phi_2(t) = -\sqrt{\frac{2}{E_g}}\,g(t)\,sin(2\pi f_c t).$$

  - $\langle \phi_1(t), \phi_2(t) \rangle = 0$ (orthogonal) under appropriate condition
  - $E_{\phi_i} = \int_{-\infty}^{\infty} |\phi_i(t)|^2\,dt = 1.$

$\Rightarrow$ orthonormal

- Then,

$$s_m(t) = \sqrt{\frac{E_g}{2}}\,cos(\theta_m)\,\phi_1(t) + \sqrt{\frac{E_g}{2}}\,sin(\theta_m)\,\phi_2(t).$$

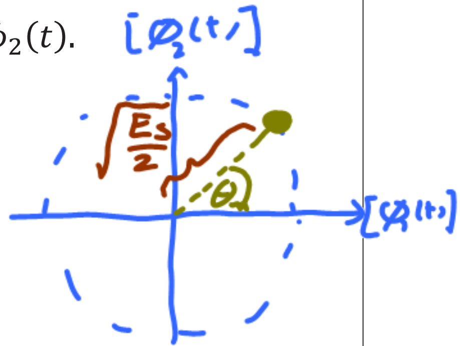
Constellations for PSK

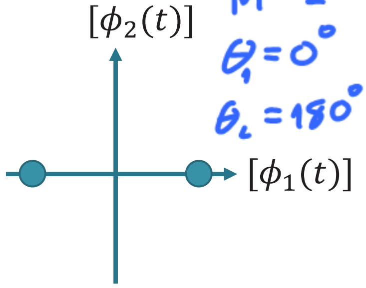
- Waveform:

$$s_m(t) = \sqrt{\frac{E_g}{2}}\,cos(\theta_m)\,\phi_1(t) + \sqrt{\frac{E_g}{2}}\,sin(\theta_m)\,\phi_2(t).$$

- Vector:

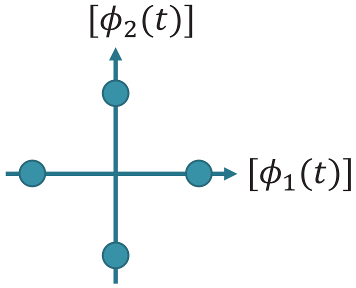
$$\vec{\mathbf{s}}^{(m)} = \left(\sqrt{\frac{E_g}{2}}\,cos(\theta_m), \sqrt{\frac{E_g}{2}}\,sin(\theta_m)\right).$$





$M = 2$: BPSK

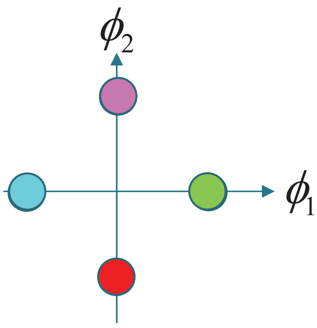
M = 2
$\theta_1 = 0°$
$\theta_2 = 180°$

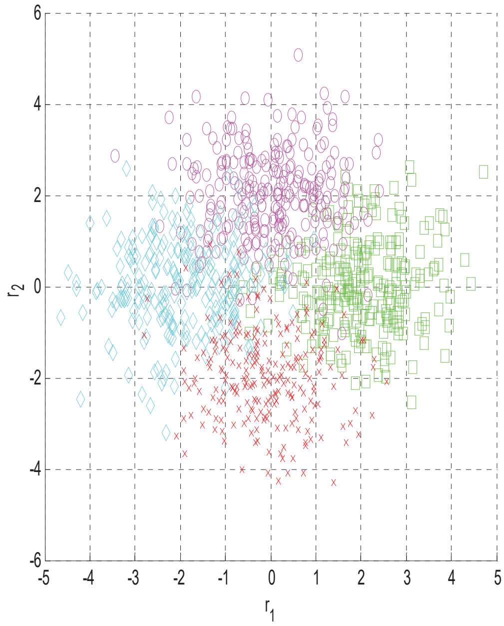


$M = 4$: QPSK

M = 4
$\theta_1 = 0°$
$\theta_2 = 90°$
$\theta_3 = 180°$
$\theta_4 = 270°$

4-PSK in the Vector Channel





$$\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$$

Quadrature Amplitude Modulation (QAM)

- Waveform:

$$s_m(t) = A_m^{(I)} g(t)\cos(2\pi f_c t) - A_m^{(Q)} g(t)\sin(2\pi f_c t), \quad m = 1,2,\ldots,M$$

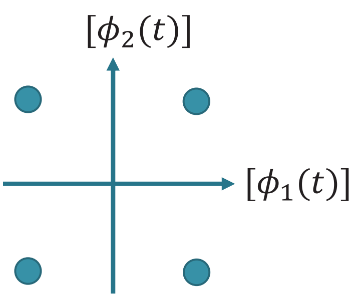
$$= A_m^{(I)}\sqrt{\frac{E_g}{2}}\,\phi_1(t) + A_m^{(Q)}\sqrt{\frac{E_g}{2}}\,\phi_2(t)$$

same defn as in PSK

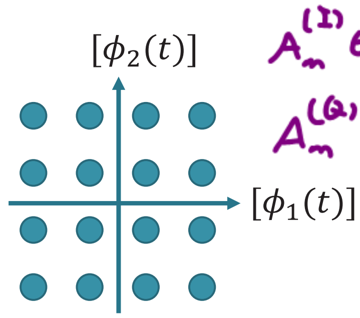
- Vector:

$$\vec{\mathbf{s}}^{(\mathrm{m})} = \left(A_m^{(I)}\sqrt{\frac{E_g}{2}}, A_m^{(Q)}\sqrt{\frac{E_g}{2}}\right).$$

Ex.
$A_m^{(I)} \in \{\pm 1\}$
$A_m^{(Q)} \in \{\pm 1\}$



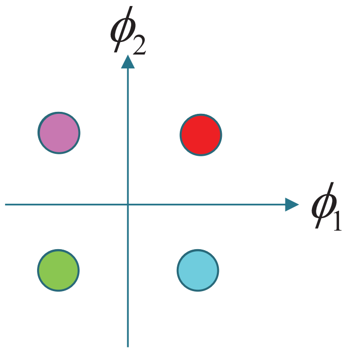
$M = 4$

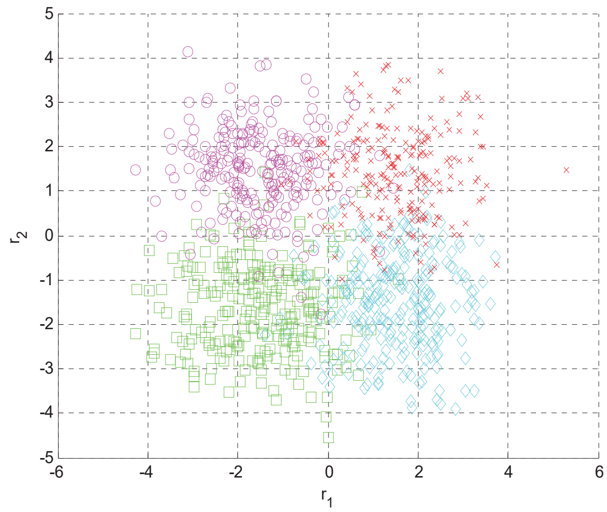


$M = 16$

$A_m^{(I)} \in \{\pm 1, \pm 3\}$
$A_m^{(Q)} \in \{\pm 1, \pm 3\}$

4-QAM in the Vector Channel





$$\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$$